

# Cournot Gaming in Joint Energy and Reserve Markets

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**Abstract**— In a traditional oligopolistic market structure, Gencos maximize profits through strategic generation offers based on the Cournot or the Supply Function equilibrium. Market power is however not limited to balancing generation and demand in real time but also to reserve in order to balance generation and demand under contingency situations. In an electricity market where online generation and spinning reserve are priced separately, hydro and thermal generators can engage in simultaneous gaming on both commodities. In this paper we examine the strategic behavior of such markets based on a Cournot-type equilibrium under which both the demand and the system reserve obey a known relationship with respect to its price. We consider the possibility of the system operator buying reserve from a continent-wide market. In addition, the Gencos' overall strategy may include selling reserve to the continental market. A number of case studies are included to demonstrate the characteristics of such a market.

**Index Terms**—Cournot, Joint energy reserve, Strategic offer, Nash equilibrium, Continental reserve market

## NOMENCLATURE

### Parameters

$C_i^0$  fixed parameter of true cost function, \$  
 $a_i^*$  first-order parameter of true cost function, \$/MWh  
 $b_i^*$  second-order parameter of true cost function, \$/MW<sup>2</sup>h  
 $a_i$  first-order parameter of offered cost function, \$/MWh  
 $b_i$  second-order parameter of offered cost function, \$/MW<sup>2</sup>h  
 $B_0$  fixed parameter of demand benefit function, \$  
 $\lambda_0$  first order parameter of demand benefit function, \$/MWh  
 $\alpha$  second-order parameter of demand benefit function, \$/MW<sup>2</sup>h

$\gamma_0$  first-order parameter of continental reserve market, \$/MWh  
 $\beta$  second-order parameter of continental reserve market, \$/MW<sup>2</sup>h

### Variables

$d$  demand level, MW  
 $g_i$  generation level of Genco  $i$ , MW  
 $g_i^{\max}$  maximum generation capacity of Genco  $i$ , MW  
 $r_i^{up}$  up spinning reserve provided by Genco  $i$ , MW  
 $r_i^{down}$  down spinning reserve provided by Genco  $i$ , MW  
 $\hat{r}_i^{up}$  allocated up reserve to be provided by Genco  $i$ , MW  
 $g_j$  largest online generation, MW  
 $\Delta d$  known largest expected decrease in demand, MW  
 $\Delta r^{up}$  reserve deficit, MW  
 $\lambda$  price of electricity, \$/MWh  
 $\gamma^{up}$  price of up reserve in continental market, \$/MWh  
 $z$  value of objective function, \$/h

### Functions

$C_i^*(g_i)$  actual generation cost function of Genco  $i$ , \$/h  
 $C_i(g_i)$  offered generation cost function of Genco  $i$ , \$/h  
 $IC_i^*(g_i)$  actual generation incremental cost function of Genco  $i$ , \$/MWh  
 $IC_i(g_i)$  offered generation incremental cost function of Genco  $i$ , \$/MWh  
 $B(d)$  demand benefit function, \$/h  
 $IB(d)$  incremental demand benefit function, \$/MWh  
 $rev_i$  revenue of Genco  $i$ , \$/h  
 $pr_i$  profit level of Genco  $i$ , \$/h

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## I. INTRODUCTION

In a traditional electricity markets, generating companies (Gencos) maximize profits through strategic generation offers. Market outcomes are supported by Nash equilibrium of strategies [1]. Gencos are assumed to either vary their generation quantity or offered price strategically or simultaneously vary both, which can be analyzed respectively by using Cournot, Bertrand and supply function equilibrium models [2][3][4].

Market power is however not limited to balancing generation and demand in real time but also to reserve in order to balance generation and demand under contingency situations. In electricity markets where online generation and spinning reserve have separate prices, hydro and thermal generators can engage in simultaneous gaming on both commodities.

In [5]-[7], authors study hybrid markets offering both energy and ancillary services and formulate tools for optimally allocating resources. However, they do not develop an optimal bidding policy. In this paper, we will show that in joint energy reserve markets, bidding strategies based only on generation are not optimal when security constraints are active. In [8], the authors develop a stochastic model and a GA to select the bid function coefficients. Competitor Genco's bidding behavior is estimated from past history. However convergence is not always guaranteed in such models.

Since generation and spinning reserve provided by any Genco are limited by its maximum generation capacity, the optimal level of both commodities as well as bidding strategies for determining optimal levels should be considered simultaneously. Hence joint bidding strategies must be considered [9].

In [10], the authors extend the joint energy reserve market to a multi zone competitive pool to allow inter-zonal reserve trading. Due to availability of transmission networks, they argue how pool-wide sharing of resources can be advantageous, as in the case of Australia. However, demand is taken to be inelastic in this model.

In this paper we examine the strategic behavior of Gencos in joint energy-reserve markets based on a Cournot-type equilibrium under which both the demand and the system reserve obey a known relationship with respect to its price. We consider the possibility of the system operator buying reserve from a continent-wide market or from the local demand. In addition, the Gencos' overall strategy may include selling reserve to the continental market. In section II, a mathematical framework has been developed and optimal Cournot-based bidding strategies are derived. Several Case studies are presented in section III.

## II. JOINT ENERGY RESERVE MARKET MODEL

Electricity Market without Reserve Constraints

We consider an electricity market with  $n$  competing generating companies (Gencos) each able to produce an amount  $g_i$  within the limits,  $0 \leq g_i \leq g_i^{\max}$  (1)

At a true cost,

$$C_i^*(g_i) = C_i^0 + a_i^* g_i + \frac{1}{2} b_i^* g_i^2, \quad (2)$$

where the parameters  $C_i^0, a_i^*, b_i^*$  are known.

The demand supplied by the  $n$  Gencos is known to have a value  $d$ , the consumption of which generates a benefit to the consumers of the form,

$$B(d) = B_0 + \lambda_0 d - \frac{1}{2} \alpha d^2, \quad (3)$$

where the parameters  $B_0, \lambda_0, \alpha$  are known.

According to the power balance between total generation and demand, we must have,

$$\sum_i g_i = d \quad (\lambda) \quad (4)$$

The Gencos game by submitting offers to supply demand which are different (higher) than their true costs,

$$C_i(g_i) \geq C_i^*(g_i) = C_i^0 + a_i^* g_i + \frac{1}{2} b_i^* g_i^2 \quad (5)$$

One possible gaming strategy is to offer according to the Cournot equilibrium, which is a relatively aggressive form of gaming.

While the profit of Genco  $i$  is,  $pr_i = \lambda g_i - C_i^*(g_i)$  and since,  $\lambda = \lambda_0 - \alpha d = \lambda_0 - \alpha \sum_j g_j$ , the increment in  $\lambda$  is then,

$$d\lambda = -\alpha dg_i.$$

Hence, taking partial with respect to  $g_i$ , the incremental cost offer becomes,

$$IC_i(g_i) = a_i^* + (b_i^* + \alpha) g_i$$

Thus, Cournot-based offers take the form,

$$C_i(g_i) = C_i^c(g_i) = C_i^0 + a_i^* g_i + \frac{1}{2} (b_i^* + \alpha) g_i^2 \quad (6)$$

Electricity Market with Reserve Constraints

When we consider security in an electricity market, generation reserve must be provided to be able to respond to contingencies such as the loss of one generator or to a sudden decrease in demand combined with high levels of wind

generation. For the first type of contingency, the Gencos offer up reserve while for the second type they offer down reserve. It is also possible for the demand-side to offer reserve, especially up reserve, by offering to curtail load voluntarily; however, for conciseness, we limit this paper to generation-supplied reserve only.

If the reserve has to compensate for the loss of the largest generator, and each Genco  $i$  provides some non-negative up reserve  $r_i^{up}$ , then the total up reserve must satisfy the inequalities,

$$\sum_i r_i^{up} \geq g_j; \forall j \quad (\gamma_j^{up}) \quad (7)$$

In addition, the up reserve of a Genco is assumed to be limited by its capacity through,

$$g_i + r_i^{up} \leq g_i^{\max}; \forall i \quad (8)$$

Similarly, each Genco  $i$  may provide some non-negative down reserve  $r_i^{down}$  defined by the inequalities,

$$\sum_i r_i^{down} \geq \Delta d \quad (\gamma^{down}) \quad (9)$$

$$\text{And, } 0 \leq g_i - r_i^{down}; \forall i \quad (10)$$

Where  $\Delta d$  is the known largest expected decrease in demand during the time interval being considered.

#### Effect of Reserve Constraints on Generation Revenues

Under reserve constraints, the Gencos submit cost offers  $C_i(g_i)$  which are not necessarily the Cournot-based offers of (6). The market however still clears by maximizing social welfare  $B(d) - \sum_i C_i(g_i)$  now subject to constraints(1), (4), as well as, (7).

The Lagrange multipliers  $\gamma^{up} = \sum_j \gamma_j^{up}, \gamma^{down}$  associated with the security requirements (7) and (9) respectively then define the marginal cost (clearing price) of up and down reserves while the Lagrange multiplier  $\lambda$  associated with the power balance (4) defines the marginal cost (clearing price) of power.

The revenue of Genco  $i$  from the sale of power as well as from the sale of up and down reserves is then,

$$rev_i = \lambda g_i + \gamma^{up} r_i^{up} + \gamma^{down} r_i^{down} \quad (11)$$

While the profit of Genco  $i$  (assuming that the cost of reserve is borne by the demand only) is,

$$pr_i = \lambda g_i + \gamma^{up} r_i^{up} + \gamma^{down} r_i^{down} - C_i^*(g_i) \quad (12)$$

If no Genco has its up and down reserve constraint (8) or (10) active then the Cournot strategy (6) will be still a valid gaming strategy. However, if either constraint (8) or (10) is active then the Cournot strategy without reserve (6) does not correspond to the Nash equilibrium since a variation in the Genco output must be matched by a corresponding variation in reserve. Thus, the Cournot strategy in an electricity market that includes both power and reserve must be reformulated as proposed next.

#### Continental Reserve Market

A possible gaming strategy that includes reserve is now considered by assuming that the electricity market is interconnected to neighbors that can supply or absorb up and down reserve deficits or surpluses according to an external ‘‘continental’’ price for such reserves. This assumes that all markets are interconnected to other markets and that they all have access to a continent-wide reserve pool.

The price of up reserve (a similar price structure applies to down reserve) is assumed to be of the form,

$$\gamma^{up} = \gamma_0 + \beta \Delta r^{up} \quad (13)$$

Where  $\gamma_0, \beta$  are known positive parameters while  $\Delta r^{up}$  is the net up reserve deficit in the electricity market defined by,

$$\Delta r^{up} = \sum_j (\hat{r}_j^{up} - r_j^{up}) \quad (14)$$

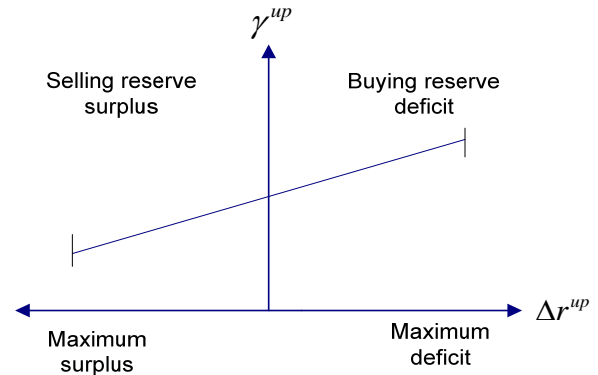


Figure 1: Price of up reserve in continental market

We can see from the figure that the continental reserve price  $\gamma^{up}$  is always positive and increases when the system buys reserve while decreasing when the system sells reserve.

We define  $\hat{r}_i^{up}$  as the up reserve ‘‘allocated’’ to Genco  $i$ , in other words, the reserve for which Genco  $i$  is responsible by either providing it or buying it from the continental market. The word ‘‘allocated’’ means that the overall up reserve imposed by security condition (7), namely the maximum

single generation output, is distributed among all Gencos in a pro-rata amount,

$$\hat{r}_i^{up} = \frac{g_i}{d} g_k, \quad (15)$$

Where  $g_k$  is the maximum generation produced by any Genco.

Thus, if the up reserve allocated to Genco  $i$ ,  $\hat{r}_i^{up}$ , is greater than the up reserve it produces,  $r_i^{up}$ , then Genco  $i$  must buy the excess  $\hat{r}_i^{up} - r_i^{up}$  in the continental up reserve market at the going price  $\gamma^{up}$ . Alternatively, if the up reserve allocated to Genco  $i$ ,  $\hat{r}_i^{up}$ , is less than the up reserve it produces,  $r_i^{up}$ , then Genco  $i$  will sell the excess  $\left| \hat{r}_i^{up} - r_i^{up} \right|$  in the continental up reserve market at the going price  $\gamma^{up}$ .

With an external reserve market, it is no longer necessary for the market Gencos to supply a minimum up reserve equal to the largest generation. The scheduled up reserve can now be above or below such a level. If above, then the market is a net seller of up reserve, if below, then the market is a net buyer of up reserve.

However with an external market, each Genco will have an incentive either to buy or sell as much as possible reserve within its generation capabilities, which means that each Genco will operate under the condition,

$$g_i + r_i^{up} = g_i^{\max} \quad (16)$$

The implication of this is that if  $r_i^{up} < \hat{r}_i^{up}$  then the Genco prefers to produce as much  $g_i$  as possible and buy its reserve deficit in the reserve market. Alternatively, if  $r_i^{up} > \hat{r}_i^{up}$  then the Genco prefers to produce as little  $g_i$  as possible in order to sell as much reserve surplus as possible in the reserve market.

### Genco Profits Including Reserve Trading and Cournot Equilibrium

The profit of Genco  $i$  including up reserve trading with the continental reserve market is (for simplicity only up reserve is treated),

$$pr_i = \lambda g_i - \gamma^{up} (\hat{r}_i^{up} - r_i^{up}) - C_i^*(g_i) \quad (17)$$

Under Cournot, we assume that only Genco  $i$  varies its output by  $dg$ , while all other Gencos hold their outputs constant. Thus, since,

$$\lambda = \lambda_0 - \alpha d = \lambda_0 - \alpha \sum_j g_j \quad (18)$$

$$\text{The increment in } \lambda \text{ is then, } d\lambda = -\alpha dg_i, \quad (19)$$

Similarly, from (13) and (14),

$$\gamma^{up} = \gamma_0 + \beta \sum_j (\hat{r}_j^{up} - r_j^{up}) \quad (20)$$

$$\text{So that, } d\gamma^{up} = \beta \sum_j (d\hat{r}_j^{up} - dr_j^{up}) \quad (21)$$

$$\text{However from (16), } dr_i^{up} = -dg_i \quad (22)$$

While from (15),

$$d\hat{r}_i^{up} = \begin{cases} \frac{dg_i}{d} g_k - \frac{g_i}{d^2} (dg_i) g_k = \left(1 - \frac{g_i}{d}\right) \frac{g_k}{d} dg_i; i \neq k \\ \frac{2dg_k}{d} g_k - \frac{g_k^2}{d^2} (dg_k) = \left(2 - \frac{g_k}{d}\right) \frac{g_k}{d} dg_k; i = k \end{cases} \quad (23)$$

So that the profit increment under Cournot for  $i \neq k$  is,

$$\begin{aligned} dpr_i &= d \left( \lambda g_i - \gamma^{up} (\hat{r}_i^{up} - r_i^{up}) - C_i^*(g_i) \right) \\ &= \left( \lambda - IC_i^*(g_i) - \alpha g_i \right) dg_i - \gamma^{up} \left( \left(1 - \frac{g_i}{d}\right) \frac{g_k}{d} dg_i + dg_i \right) \\ &\quad - \beta (\hat{r}_i^{up} - r_i^{up}) \left( \left(1 - \frac{g_i}{d}\right) \frac{g_k}{d} dg_i + dg_i \right) \end{aligned} \quad (24)$$

Equating to zero, gives the Cournot strategy with up reserve for Genco  $i \neq k$ , that is,

$$\begin{aligned} IC_i(g_i) &= IC_i^*(g_i) + \alpha g_i \\ &\quad + \left( \gamma^{up} + \beta (\hat{r}_i^{up} - r_i^{up}) \right) \left( \left(1 - \frac{g_i}{d}\right) \frac{g_k}{d} + 1 \right) \end{aligned} \quad (25)$$

Similarly, we can derive an equation for Genco  $i = k$ . From (25), we see that the first two terms correspond to the Cournot strategy of Genco  $i$  without reserve. The consideration of reserve adds a new term to the gaming strategy with two components.

From (17), setting  $\frac{dpr_i}{dg_i} = 0$ , the optimal Genco offered cost function will be of the form,

$$\begin{aligned} C_i(g_i) &= C_i^*(g_i) + \frac{\alpha g_i^2}{2} \\ &\quad + \left( \gamma_0 + \beta \sum_j (\hat{r}_j^{up} - r_j^{up}) \right) (\hat{r}_i^{up} - r_i^{up}) \end{aligned} \quad (26)$$

### Market Clearing in Presence of Continental market

To obtain the market outcome considering a continental market and assuming Gencos offer according to Cournot-based strategy (26), the ISO can maximize the value of the objective function,

$$B(d) - \sum_i C_i(g_i) - \gamma^{up} \sum_i (\hat{r}_i^{up} - r_i^{up}), \text{ subject to}$$

constraints (4), (15) and (16).

### III. CASE STUDIES

- Market clearing schemes for comparison:
  - Case A: No reserve (internal or external) constraints are active.
  - Case B: We assume Gencos do not bid optimally. Only internal up spinning reserve is considered.
  - Case C: Gencos bid according to Cournot-based strategy (6), which does not take into account the impact of reserve constraints. Only internal up spinning reserve is considered.
  - Case D: Gencos do not bid optimally. Interaction is possible with a continental market for up reserve. System operator can buy reserve deficit or Gencos can sell reserve surplus at continental market.
  - Case E: Gencos bid optimally according to Cournot-based strategy (26) at presence of continental market
- Effect of varying continental reserve market parameters

The parameters for Genco cost functions and Genco capacity is captured in Table I. Parameters describing characteristics of demand elasticity and continental reserve market is captured in Table II.

Table I: Genco parameters

	G1	G2	G3
Cost function parameter, $a_i^*$ \$/MWH	20	30	40
Cost function parameter, $b_i^*$ \$/MW <sup>2</sup> H	0.05	0.05	0.05
$G_i^{\max}$ MW	800	500	500

Table II: Demand benefit and external reserve market parameters

Demand benefit parameter, $\lambda^0$ \$/MWH	150
Demand benefit parameter, $\alpha$ \$/MW <sup>2</sup> H	0.05
Continental reserve market parameter, $\gamma_0$ \$/MWH	23.3
Continental reserve market parameter, $\beta$ \$/MW <sup>2</sup> H	0.001

We have used commercial optimization solver MINOS (DNLP) in GAMS environment. The market outcomes of different schemes are presented in Table III.

Table III: Comparing different economic dispatch schemes

		A	B	C	D	E
		No gaming, No reserve	No gaming, internal reserve	Gaming on $g_i$ , only internal reserve	No Gaming, continental market	Gaming, continental market
Generation levels, MW	$g_1$	800	466.7	485.7	500	389.4
	$g_2$	500	466.7	464.3	500	389.4
	$g_3$	450	400	364.3	365.6	296.5
Load served, MW	$d$	1750	1333.3	1314.3	1365.6	1075.2
Allocated reserve, MW	$r_1^*$	N/A	N/A	N/A	183.1	140.9
	$r_2^*$	N/A	N/A	N/A	183.1	140.9
	$r_3^*$	N/A	N/A	N/A	133.9	107.7
Dispatched reserve, MW	$r_1$	0	333.3	314.3	300	410.6
	$r_2$	0	33.3	35.7	0	110.6
	$r_3$	0	100	135.7	134.4	203.5
Reserve deficit, MW	$\Delta r$	N/A	N/A	N/A	65.6	-335.4
Energy price, \$/MWH	$\lambda$	62.5	83.3	84.3	90.3	99.759
Continental reserve Price, /MWH	$\gamma^{up}$	N/A	23.3	7.9	23.4	22.965
Profits, \$/H	$pr_1$	17900	31788.9	27695.9	31527.7	33357.1
	$pr_2$	9800	20022.2	19895.7	19417.9	22474.2
	$pr_3$	4862.5	15466.7	13681.4	14857.4	17528.3
Load profit \$/H		76712	33705	39517	35067	25268
SW \$/H	$z$	109275	100983.3	86185.7	100870	88849.6

From table III we notice that,

- In case B, due to inclusion of reserve constraints, total load served decreases compared to case A.
  - In case C, when Gencos bid strategically according to Cournot (6) without considering impact of reserve constraints, maximum social welfare degrades. Their profits do not improve, since bidding strategies were not optimal.
  - In case D, by introducing continental reserve market, cheaper Gencos provide more generation and total load served and maximum SW improve significantly. Gencos' profits do not change considerably compared to case B. Since the system is a net buyer of reserve deficit, the continental price of reserve increases.
  - In case E, when Gencos bid according to Cournot-based strategy (26), load served and max SW drastically reduce. Gencos' profits improve. The system is a net seller of reserve surplus, so price of reserve decreases in continental market.
- Effect of varying continental reserve market parameters

Next, we vary the continental reserve market parameters to see the effect on the market outcomes. Results are summarized in Table IV.

Table IV: Effect of varying continental reserve market parameters

Cournot gaming at presence of continental reserve market		$\gamma_0 = 23.3,$ $\beta = 0.001$	$\gamma_0 = 15,$ $\beta = 0.001$	$\gamma_0 = 23.3,$ $\beta = 0.01$
Generation levels, MW	$g_1$	389.4	427.3	414.0
	$g_2$	389.4	427.3	414.0
	$g_3$	296.4	350.7	313.3
Load served, MW	$d$	1075.2	1205.3	1141.4
Allocated reserve, MW	$r'_1$	140.9	151.5	150.1
	$r'_2$	140.9	151.5	150.1
	$r'_3$	107.3	124.3	113.6
Dispatched reserve, MW	$r_1$	410.6	372.6	385.9
	$r_2$	110.6	72.6	85.9
	$r_3$	203.5	149.3	186.6
Reserve Deficit, MW	$\Delta r$	-335.4	-167.3	-244.5
Energy Price, \$/MWH	$\lambda$	99.7	92.6	96.4
External Reserve Price, \$/MWH	$\gamma^{up}$	22.9	14.8	20.8
Profits, \$/H	$pr_1$	33357.1	29639.4	32200
	$pr_2$	22474.2	20816.1	21703.4
	$pr_3$	17528.3	15541.5	16568.1
Load Profit \$/H		25268	33016	28658
SW \$/H	$z$	88849.6	86808.2	88104.3

From Table IV we can summarize the effect of varying continental reserve market parameters as follows:

- If  $\gamma_0$  is high, Gencos improve profits by selling more reserve in continental market, thus system offers more reserve surplus.
- With lower  $\gamma_0$ , cheaper Gencos provide more generation than reserve, hence total load served is higher.
- Max SW does not vary significantly.
- With lower  $\gamma_0$ , load profit is higher, but Genco profits are lower.

#### IV. CONCLUSIONS

Gencos may maximize profits not only based on strategic generation offers, but also on reserve offers. We have formulated a joint energy reserve market model where an ISO can interact with a continental market for trading reserve deficit or surplus. This can maximize utilization of transmission lines. Deterministic security constraints have been considered. We have developed the optimum Cournot-incremental cost offer strategy for Gencos offering only generation and up spinning reserve and where demand and continental reserve follow a known relationship with respect to price. A number of examples have been presented to illustrate how Gencos interact with a continental market and offer generation and spinning reserve strategically to maximize profits.

We have not considered transmission charges in this paper. However, a multi-area inter connected model is currently under investigation, where transmission charges and congestion in tie lines will be taken into account. Moreover, the model can be extended to allow demand side participation to offer up spinning reserve by strategically curtailing load.

Other future research could include stochastic security, SFE approach to analyze agents' strategic behaviors, hybrid deterministic-stochastic model to allocate reserve at presence of significant wind generation.

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## VI. BIOGRAPHIES

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